On the generalization of the Darboux theorems

Kaveh Eftekharinasab
(Institute of mathematics of NAS of Ukraine)
E-mail: `kaveh@imath.kiev.ua`

We refer to [1] for the definitions concerning the category of MC^k-Frechet manifolds.
We prove that vector fields have local flows.

Theorem 1. *Let F be a Fréchet space, X an MC^k-vector field on $U \subset F$, $k \geqq 1$. There exists a real number $\alpha > 0$ such that for each $x \in U$ there exists a unique integral curve $\ell_x(t)$ satisfying $\ell_x(0) = x$ for all $t \in I = (-\alpha, \alpha)$. Furthermore, the mapping $\mathbb{F} : I \times U \to F$ given by $\mathbb{F}_t(x) = \mathbb{F}(t, x) = \ell_x(t)$ is of class MC^k.*

Therefore we are able to apply Moser's approach, that is constructing an appropriate isotopoy generated by a time dependent vector field that provides the chart transforming of symplectic forms to constant ones to prove the Darboux theorem in the category of MC^k-manifolds.

Definition 2. Let M be a bounded Fréchet manifold. We say that M is weakly symplectic if there exists a closed smooth 2-form ω such that it is weakly non-degenerate i.e. for all $x \in M$ and $v_x \in T_xM$

$$\omega_x(v_x, w_x) = 0 \tag{1}$$

for all $w_x \in T_xM$ implies $v_x = 0$.

Let F'_b be the strong dual of F and define the map $\omega^\#_x : F \to F'_b$ by

$$\langle w, \omega^\#_x(v)\rangle = \omega_x(w, v),$$

where $\langle \cdot, \cdot \rangle$ is a duality pairing. Condition 1 implies that $\omega^\#$ is injective.

Let $x \in U$ be fixed and define $H_x = \{\omega_x(y, .) \mid y \in F\}$, this is a subset of F'_b and its topology is induced from it. We assume that all Fréchet spaces are reflexive.

Lemma 3. *$\omega^\#_x : F \to H_x$ is an isomorphiam.*

Theorem 4. *Let (M, ω) be a weakly symplectic smooth bounded Fréchet manifold modeled on F. Let $\omega^t = \omega_0 + t(\omega - \omega_0)$ for $t \in [0, 1]$. Suppose that following hold*

(1) There exits an open star-shaped neighborhood $\mathcal{U}$ of zero such that for all $x \in \mathcal{U}$ the map $\omega^{t\#}_x : F \to H_x$ is isomorphism for each t,

(2) for $x \in \mathcal{U}$ the map $(\omega^{t\#}_x)^{-1} : H_x \to E$ is smooth for each t.

Then there exists a coordinate chart $(\mathcal{V}, \varphi)$ around zero such that $\varphi^*\omega = \omega_0$.

References

[1] Kaveh Eftekharinasab. Geometry of bounded Fréchet manifolds, *Rocky Mountain Journal of Mathematics*, 46(3) : 895–913, 2016.